

Kruskal W H & Wallis W A. Use of ranks in one-criterion variance analysis. *J. Amer. Statist. Assn.* 47:583-621, 1952; and Errata. *J. Amer. Statist. Assn.* 48:907-11, 1953.
[Committee on Statistics, University of Chicago. IL]

A test is proposed for the null hypothesis that several independent samples arise from the same population. All of the observations are ranked, and a test statistic based on the ranks (and a simple function of the conventional analysis of variance test statistic applied to the ranks) is computed. The approximate null distribution for the test statistic is a familiar, well-tabulated chi-square distribution. [The *SCJ*® and *SSCJ*® indicate that these papers have been cited in over 565 publications since 1955.]

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Our paper on one-criterion variance analysis was written soon after W.A. Wallis, together with the late L.J. Savage, founded the Committee on [later Department of] Statistics at the University of Chicago. W.H. Kruskal had recently come to Chicago from graduate studies at Columbia University.

The idea of doing a standard analysis of variance on ranks was natural at the time, and in retrospect it is surprising that others had not published the details for the one-way case. Ranks have a long statistical history, in which a landmark was the 1936 paper by H. Hotelling and M.R. Pabst.¹ A second strand was that of permutation tests, proposed by R.A.

Fisher for a different purpose, and worked out in various settings by E.J.G. Pitman and B.L. Welch. In 1937, Milton Friedman, a close colleague of Wallis, published the analogous rank procedure for the two-way structure with one observation per cell;² in 1945 and 1947 respectively, F. Wilcoxon³ and H.B. Mann and D.R. Whitney⁴ published the two-sample procedure that we generalized to more than two samples. (Rediscoveries are described in our paper.) Furthermore, Wallis in 1939 presented the rank-based version of the correlation ratio.⁵

So in a sense our 1952 paper, along with the associated 1952 paper by Kruskal alone,⁶ made solid what was already in the air; it also provided mathematical underpinnings and discussed such important side matters as the adequacy of the asymptotic distribution, exact significance levels for small sample sizes, and other relevant procedures in the literature. The basic idea was Wallis's; Kruskal provided some theoretical calculations, and both of us worked on side matters.

No doubt a large fraction of the citations to our paper were made in a gracious documentary spirit along with applications of the procedure, which was quickly incorporated into the statistical armamentarium. Other citations were undoubtedly made by theoretical statisticians striving for better understandings of the procedure (for example, of its power), showing how it is a special case of generalizations, or working out variants (for example, for alternative hypotheses in which the subpopulations are stochastically ordered; again, for multiple comparison procedures).

Aspects of our procedure are treated in the 1978 *International Encyclopedia of Statistics*, especially by Herbert A. David⁷ and Peter Nemenyi,⁸ and in the 1983 *Encyclopedia of Statistical Sciences* by Helen Bhattacharyya.⁹ In his magisterial text on nonparametrics, E.L. Lehmann discusses our procedure in detail.¹⁰

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