

Dyson F J. General theory of spin-wave interactions.

Phys. Rev. 102:1217-30, 1956.

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A spin-wave is a collective motion of the atomic spins in the Heisenberg model of a ferromagnet. The main result of this paper is a precise calculation of the interaction between two spin-waves in the limit of long wavelengths or low temperatures. [The *SCI*® indicates that this paper has been cited in over 675 publications.]

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During the 1950s, I used to spend summers working as a postdoc in Charles Kittel's group in Berkeley. Each June when I arrived in Berkeley, Kittel would have a problem ready for me, and I would spend the summer solving it. The problem for 1955 was to resolve a paradox that had arisen in the theory of ferromagnets. On the one hand, the old linear spin-wave theory of Bloch agreed well with experiments.¹ (p. 360-3) On the other hand, the coupling between spins in the Heisenberg model is strong and nonlinear, and various more recent theoretical estimates of the effects of spin-coupling had predicted strong deviations from the Bloch theory.² I was able to resolve the paradox by calculating accurately the nonlinear effects and proving that they were too small to be observable. The basic reason why the nonlinear effects are small is that spin-waves of long wavelength float over each other like long waves on the surface of the ocean.

The essential difficulty in constructing an accurate theory of spin-waves is the mismatch

between the discrete structure of an atomic ferromagnet and the continuous structure of the spin-waves. The atomic description and the spin-wave description of the ferromagnet are supposed to be equivalent, but the spin-waves have more degrees of freedom than the atoms. The spin-wave states are, in fact, redundant. The main technical problem that I solved was to work out a consistent method for handling redundant states.

The practical consequence of my analysis was merely to confirm what Kittel already knew, that the simple Bloch model of spin-waves was good enough for everyday use. The result of my paper was nothing new. What was new was the mathematical machinery. The paper became famous because it introduced for the first time into solid-state physics the heavy apparatus of Green's Functions and higher-order perturbation theory borrowed from quantum electrodynamics. The apparatus of quantum field-theory was subsequently exploited by other condensed-matter physicists and turned out to be particularly useful in the development of theories of superfluidity and superconductivity. The application to spin-waves was only a preliminary exercise.

My paper was superseded and made obsolete by the brilliant work of Zittartz³ nine years later. Zittartz replaced my cumbersome and obscure mathematics by a simple and elegant construction. He obtained all my results in a few pages with a minimum of calculation. I do not know why it has happened that Zittartz's paper is unjustly neglected while mine is still so widely cited. Probably the main reason is that the definitive review article on spin-waves was published by Frederic Keffer in 1966,⁴ just too late to include a reference to Zittartz. Keffer happened to be a student of Kittel and gave my paper a disproportionately prominent place in his review. After 1966, the subject of spin-wave interactions went into a long sleep, and so Keffer's review became effectively the judgment of history.

1. Kittel C. *Introduction to solid state physics*. New York: Wiley, 1953. 396 p.
2. Van Kranendonk J. Spin-deviation theory of ferromagnetism. II. The non-ideal spin-deviation gas. *Physica* 21:925-45, 1955.
3. Zittartz J. On the spin-wave problem in the Heisenberg model of ferromagnetism. *Z. Phys.* 184:506-20, 1965. (Cited 1 time.)
4. Keffer F. Spin waves. *Handbuch der Physik* 18(Pt.2):1-273, 1966. (Cited 575 times.)